

Hadron spectroscopy of twisted mass lattice QCD at $\beta = 6.0$. *

UKQCD Collaboration, Craig McNeile and Chris Michael,^a

^aDepartment of Mathematical Sciences, University of Liverpool, L69 3BX, UK

Simulations that use the clover action in quenched QCD calculations have a lower limit to the quark mass that can be reached, because of the fluctuations caused by exceptional configurations. From this low statistics study, we find that the twisted clover action, recently introduced by the ALPHA collaboration, can be used to simulate quenched QCD at quark masses below those attainable by simulations that use the clover action.

1. INTRODUCTION

The masses of the so called light quarks used in lattice QCD calculations are not light enough for chiral perturbation theory to be a reliable guide to extrapolating the results to the physical quark masses. For example, in our recent work on determining the parameters of the chiral QCD Lagrangian [1], we were unable to test fully the chiral perturbation theory predictions because the dynamical quarks were too heavy and there was a lower limit on the mass of the valence quarks caused by exceptional configurations.

The ALPHA collaboration have developed [2] the twisted clover fermion action, specifically to study the light quark mass region of QCD. The ALPHA collaboration [3] have used the twisted clover action in numerical simulations within the Schrödinger functional formalism. We report the first computation of hadron spectroscopy using the twisted clover fermion operator with periodic in space and anti-periodic in time boundary conditions.

2. THE TWISTED-FERMION OPERATOR

$$S_F = \sum_x \bar{\psi}(x)(D + m_q + i\mu_q\gamma_5\tau_3)\psi(x) \quad (1)$$

The operator D is the fermion operator that is independent of the quark mass. The mass term

$$i\mu_q\gamma_5\tau_3 = i\mu_q\gamma_5 \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad (2)$$

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has no additive renormalisation, hence it provides a lower limit to the eigenvalue spectrum of the fermion operator.

The main disadvantages of the twisted clover action are that parity and flavour symmetry are explicitly broken. However, the flavour symmetry breaking looks very different to the mechanism in the staggered fermion formulation [4], so the effects of the flavour symmetry breaking may be less of a problem.

The transformation

$$\psi' \rightarrow e^{i\alpha\gamma_5\tau_3/2}\psi \quad \bar{\psi}' \rightarrow \bar{\psi}e^{i\alpha\gamma_5\tau_3/2} \quad (3)$$

transforms the action in Eq. 1, when D anti-commutes with γ_5 as in the continuum, into the standard QCD action

$$S_F = \sum_x \bar{\psi}'(x)(D + M_Q)\psi'(x) \quad (4)$$

where

$$M_Q = \sqrt{m_q^2 + \mu_q^2}$$

$$\tan \alpha = \frac{\mu_q}{m_q}$$

The ALPHA collaboration [2,5] advocate the following procedure. The simulations are carried out using the twisted clover operator on the lattice. The results are then matched to the continuum twisted fermion operator. The rotations in Eq. 3 are then used to rotate back to the standard QCD fermion action in the continuum. The rotations should reduce the effect of the flavour and parity symmetry breaking on the physical spectrum. The connection between QCD and twisted

QCD could be more complicated, when there is spontaneous breakdown of parity and flavour symmetry, such as in the Aoki phase [6].

Another way to look at the connection between the twisted and the traditional clover action is to think that the rotations Eq. 3 are done on the lattice [7]. In this viewpoint the rotations (Eq. 3) are similar to the effect of a preconditioner, except that the connection between the twisted and standard clover actions is not exact, because the transformation in Eq. 3 also rotates the Wilson term in the operator,

The formalism for the $O(a)$ improvement of the twisted fermion operator has been described in [5].

3. INTERPOLATING OPERATORS

The parity and flavour symmetry violating parts of the twisted clover action make the construction of the interpolating operators nontrivial. ALPHA [2] use the twisting rotations in Eq. 3 to construct the interpolating operators.

Consider a general meson interpolating operator

$$\bar{\psi}\Gamma\frac{\tau_i}{2}\psi \quad (5)$$

For the Wilson/clover action all three meson operators ($i = 1,2,3$) are equivalent. The flavour symmetry breaking term in the twisted action breaks the equivalence [6].

For $i = 1,2$ the generic meson operators are

$$\begin{aligned} \bar{u}\Gamma d' &= \bar{u}(\cos(\alpha/2) + i\gamma_5 \sin(\alpha/2)) \\ \Gamma &(\cos(\frac{\alpha}{2}) - i\gamma_5 \sin(\frac{\alpha}{2}))d \end{aligned} \quad (6)$$

For $\Gamma = 1$ (scalar), γ_5 (pion), $\gamma_i\gamma_j$ (b_1) and $\gamma_4\gamma_i$ (second rho operator).

$$\bar{u}\Gamma d' = \bar{u}\Gamma d \quad (7)$$

For $\Gamma = \gamma_4\gamma_5$ (pion), γ_4 ("exotic"), γ_i (rho)

$$\bar{u}\Gamma d' = \cos(\alpha)\bar{u}\Gamma d + i\sin(\alpha)\bar{u}\gamma_5\Gamma d \quad (8)$$

The neutral meson operator is proportional to

$$\bar{u}\Gamma u - \bar{d}\Gamma d \quad (9)$$

The neutral meson operator has the opposite connection between the gamma matrix and the mixing as for the charged meson operators. Disconnected fermion loops are required to compute the correlators for the neutral meson operator, because the twisted fermion operator does not obey $M^\dagger = \gamma_5 M \gamma_5$ [6].

To compute the correlator for the nucleon with an interpolating operator, such as:

$$P(x)_i = \epsilon_{abc}(u^a(x)^T C \gamma_5 d^b(x))u_i^c(x), \quad (10)$$

separate inversions are required for the up and down twisted clover propagators. These can be rotated back to the clover fermion matrix using Eq. 3, before the nucleon correlator is constructed.

4. NUMERICAL RESULTS

The simulations were performed at $\beta = 6.0$ on $16^3 48$ lattice in the quenched approximation. The numerical value of the clover coefficient determined by the ALPHA collaboration [8] was used. The UKQCD collaboration [9] found, at the same parameters, evidence for exceptional configurations at $M_{PS}/M_V \sim 0.54$, hence we aimed to use the twisted clover action to explore the quark mass region under that limit. For this exploratory study we only used 30 configurations.

UKQCD found [9] $\kappa_{critical} = 0.135252$, so we initially ran at $(\kappa = 0.135, \mu = 0.02)$, $(\kappa = 0.135, \mu = 0.01)$, and $(\kappa = 0.135, \mu = 0.005)$. We were aiming for $\alpha \sim \frac{\pi}{2}$, but this κ value was not close enough to $\kappa_{critical}$, and the α values for the runs $\mu = 0.02, 0.01$, and 0.005 were: $0.41\pi, 0.33\pi, 0.24\pi$. For all these runs we used the standard stabilised biconjugate gradient algorithm. To increase α to be close to $\frac{\pi}{2}$ we ran at $(\kappa = 0.13525, \mu = 0.01)$ and $(\kappa = 0.13525, \mu = 0.005)$. For the inversions with $\kappa = 0.13525$, the conjugate gradient algorithm had to be used.

With the low statistics we could only obtain satisfactory fits to the pion correlator. Factorising fits [9] were performed to a 2 by 2 matrix, with a basis of fuzzed and local correlators [9]. The pion mass as a function of the renormalised quark mass (M_Q in Eq. 4 [2], with μ_q/Z_P and $m_q = Z_A m_{pcac}/Z_P$, where m_{pcac} is the mass from

the PCAC relation) is plotted in Fig. 1. The pion mass from the twisted clover simulation with the parameters $\kappa = 0.13525, \mu = 0.005$ is $\sim 60\%$ lower than the pion mass obtained from the clover action result at the point where exceptionals become a problem ($\kappa = 0.13455$).

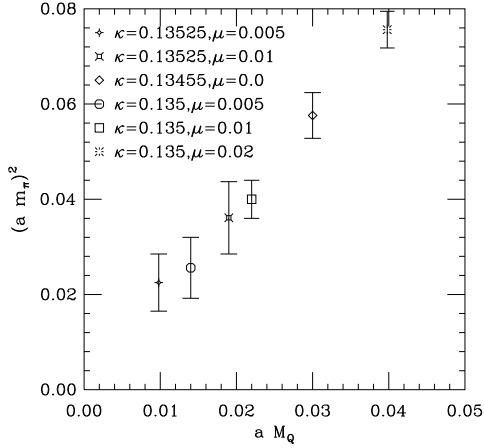


Figure 1. The pion mass squared as a function of renormalised quark mass.

5. ANALYSIS OF THE EIGENVALUES

It is instructive to consider the effect of the γ_5 term on the eigenvalue spectrum of the clover fermion matrix. Consider the eigenvalue equation for a single flavour of the twisted clover action.

$$(M + i\mu\gamma_5)x(\mu) = \lambda(\mu)x(\mu) \quad (11)$$

where M is the fermion matrix of the clover action. The relation $M^\dagger = \gamma_5 M \gamma_5$ implies that the eigenvalues of M are either real, or come in complex conjugate pairs (see [10] for a review).

The real eigenvalues of M are also eigenvalues of γ_5 , hence the eigenvectors of the clover fermion matrix with real eigenvalues are also eigenvectors of the twisted matrix with complex eigenvalues.

The complex eigenvalues of M are not eigenvalues of γ_5 , so there is no simple connection between the eigenvalues of the twisted clover fermion operator and those of the clover fermion operator. Perturbation theory shows that the first order

perturbation [11] of the $i\gamma_5\mu$ term on the spectrum is

$$\left| \frac{d\lambda(\mu)}{d\mu} \right| = \left| \frac{y^\dagger \gamma_5 x}{y^\dagger x} \right| \quad (12)$$

where y is the left eigenvector of the clover fermion operator M . Unfortunately the symmetries of the clover operator M do not constrain the matrix element of $y^\dagger \gamma_5 x$. The complex eigenvalues of M are changed at $O(\mu)$ by the $i\mu\gamma_5$ term.

We are using the ARPACK package to numerically study the eigenvalues of the twisted clover action.

6. CONCLUSIONS

In the quenched approximation, the twisted clover action has allowed us to simulate at lighter pion masses than could be reached by the clover action. We are increasing the statistics at the lightest quark mass.

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